ගැටළු - 01

01. $y = e\sin^{-1}x$ නම්, $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$ බව පෙන්වන්න.

02. $y = 4(\sin^{-1}x)^2$ නම්, $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 8 = 0$ බව පෙන්වන්න.

පිළිතුරු - 01

01. $y = e\sin^{-1}x$ ———(1) දෙපසම x විෂයෙන් අවකලනය,

$$\frac{dy}{dx} = \underbrace{e\sin^{-1}x}_{=y}\ \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\ \frac{dy}{dx} = y \quad \left[\because (1)\right] \text{———(2) දෙපසම වර්ග කරමු.}$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = y^2 \text{ දෙපසම } x \text{ විෂයෙන් අවකලනය,}$$

$$(1-x^2)\cancel{2}\ \frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\cancel{2}}(-\cancel{2}x) = \left(\frac{dy}{dx}\right)\cancel{2}y$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0 \ //$$

02. $y = 4(\sin^{-1}x)^2$ ———(1)

$$\frac{dy}{dx}\ 4\ 2\sin^{-1}x\ \frac{1}{\sqrt{1-x^2}} \text{ දෙපසම වර්ග කරමු.}$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 64(\sin^{-1}x)$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 16y \left[\because (1)\right] \text{ දෙපසම } x \text{ විෂයෙන් අවකලනය,}$$

$$(1-x^2)\cancel{2}\left(\frac{dy}{dx}\right)\ \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\cancel{2}}(-\cancel{2}x) = \cancel{16}^{8}\ \frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 8 = 0 \ //$$

ගැටළු - 02

03. $y = e^{3x} \sin 4x$ නම්, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$ බව පෙන්වන්න.

04. $y = \ln(\cot x)$ නම්, $\tan 2x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ බව පෙන්වන්න.

පිළිතුරු - 02

03. $y = e^{3x} \sin 4x$ ——— (1)

$$\frac{dy}{dx} = e^{3x} \cos 4x.4 + \underbrace{\sin 4x.e^{3x}.3}_{=y}$$

$$\frac{dy}{dx} = 4e^{3x} \cos 4x + 3y \text{ ——— (2)}$$

$$\frac{d^2y}{dx^2} = 4 . \left[e^{3x}(-\sin 4x) . 4 + \cos 4x . e^{3x} . 3\right] + 3\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \underbrace{-16\, e^{3x} \sin 4x}_{=y} + 12\, e^{3x} \cos 4x + 3\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -16y + 3 . 4e^{3x} \cos 4x + 3\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -16y + 3 . \left(\frac{dy}{dx} - 3y\right) + 3\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -6\frac{dy}{dx} + 25y = 0 \;//$$

04. $y = \ln(\cot x)$ ——— (1)

$$\frac{dy}{dx} = \frac{1}{\cot x} . \operatorname{cosec}^8 x$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos x} . \frac{1}{\sin^2 x}$$

$$\frac{dy}{dx} = \frac{1.2}{2 \sin x \cos x}$$

$$\sin 2x \frac{dy}{dx} = -2$$

$$\sin 2x \frac{d^2y}{dx^2} + \frac{dy}{dx} \cos 2x - 2 = 0$$

$\div \cos 2x$

$$\tan 2x \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2 = 0 \;//$$

ගැටළු - 03

05. $y = a\cos(\ln x) + b\sin(\ln x)$ නම්, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ බව පෙන්වන්න.

06. $y = 1 + x + \frac{x^2}{2!} + \ldots\ldots\ldots + \frac{x^{2n}}{2n!}$ විට $\frac{dy}{dx} + \frac{x^{2n}}{2n!} = y$ බව පෙන්වන්න.

$x^2\frac{d^2y}{dx^2} - (2n + x)\frac{dy}{dx} + 2ny = 0$ බව පෙන්වන්න.

07. $x = a(\sin t + \cos t)$ ද $y = a(\sin t - \cos t)$ ද විට $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ බව පෙන්වන්න.

පිළිතුරු - 03

05. $y = a\cos(\ln x) + b\sin(\ln x)$

$x\frac{dy}{dx} = -a\sin(\ln x) + b\cos(\ln x)$

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - a\cos(\ln x) - b\sin(\ln x)$

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ //

06. $y = 1 + x + \frac{x^2}{2!} + \ldots\ldots\ldots + \frac{x^{2n}}{2n!}$ විට

$\frac{dy}{dx} = 1 + x + \frac{x^2}{2!} + \ldots\ldots\ldots + \frac{x^{2n-1}}{(2n-1)}$

$\frac{dy}{dx} = y - \frac{x^{2n}}{2n!}$

$\therefore \frac{dy}{dx} + \frac{x^{2n}}{2n!} = y$ //

$\frac{d^2y}{dx^2} + \frac{x^{2n-1}}{2n-1!} = \frac{dy}{dx}$

$x\frac{d^2y}{dx^2} + \frac{x^{2n-1}}{2n-1!} = x\frac{dy}{dx}$

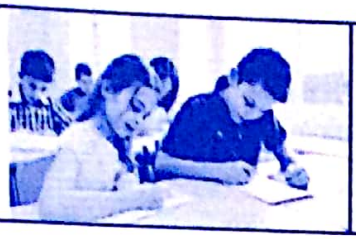
$x\frac{d^2y}{dx^2} + 2n\left(-\frac{dy}{dx} + y\right) = x\frac{dy}{dx}$

$\therefore x^2\frac{d^2y}{dx^2} - (2n + x)\frac{dy}{dx} + 2ny = 0$ //

07. $\frac{dy}{dx} = a(\cos t - \sin t)$, $\frac{dy}{dx} = a(\cos t + \sin t)$

$\frac{dy}{dx} = -\frac{x}{y}$

$\therefore y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ //



ගැටළු

01. $\log_e |x^2 + y^2| = 4 \tan^{-1}\left(\frac{y}{x}\right)$ වේ. $2x \neq y$ විට $\frac{dy}{dx} = \frac{x+2y}{2x-y}$ බව පෙන්වා එනයින් $\frac{d^2y}{dx^2}$ $\frac{5(x^2+y^2)}{(2x-y)^3}$ බව පෙන්වන්න.

02. $y = e^{2m\cos^{-1}x}$ නම්, $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 4m^2 y = 0$ බව පෙන්වන්න.

03. m නියත විට $y = e^{m\tan^{-1}x^2}$ නම් $(1+x^4)\frac{dy}{dx} = 2mxy$ බව පෙන්වා $x = 1$ විට $\frac{dy}{dx}$ අගය ලබාගන්න.

එනයින් $(1+x^4)\frac{d^2y}{dx^2} + 2x(2x^2 - m) - 2my = 0$ බව පෙන්වන්න. $m = 4$ විට $\left(\frac{d^2y}{dx^2}\right)_{x=1}$ අගයද ලබාගන්න.

04. (i) $y = x\cos\frac{1}{x}$ නම් $x^4\frac{d^2y}{dx^2} + y = 0$ බව පෙන්වන්න.

(ii) x හා y යනු $x\cos^4 t$, $y = \sin^4 t$ ලෙස t පරාමිතියෙන් අර්ථ දක්වා ඇත. t ඇසුරින් $\frac{dy}{dx}$ ද, $\frac{d^2y}{dx^2}$ ද සොයන්න. එනයින් $2\frac{d^2y}{dx^2} - \left(1 - \frac{dy}{dx}\right)^3 = 0$ බව පෙන්වන්න.

පිළිතුරු

01. $\log_e |x^2 + y^2| = 4 \tan^{-1}\left(\frac{y}{x}\right)$ x විෂයයෙන් අවකලනයෙන්,

$$\frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right) = 4 \times \frac{1}{1+\left(\frac{y}{x}\right)^2}\left[\frac{x\frac{dy}{dx} - y \times 1}{x^2}\right]$$

$$\frac{2\left(x + y\frac{dy}{dx}\right)}{x^2+y^2} = \frac{4x^2}{x^2+y^2} \times \frac{\left(x\frac{dy}{dx} - y\right)}{x^2}$$

$$x^2 + y^2\frac{dy}{dx} = 2\left(x\frac{dy}{dx} - y\right)$$

$$\frac{dy}{dx}(2x - y) = x + 2y \quad \text{——(1)}$$

$$\frac{dy}{dx} = \frac{x+2y}{2x-y}$$

(1), x විෂයයෙන් අවකලනයෙන්,

$$\frac{d^2y}{dx^2}(2x-y) + \frac{dy}{dx}\left(2 - \frac{dy}{dx}\right) = 1 + 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}(2x-y) = 1 + \left(\frac{dy}{dx}\right)^2$$

$\frac{dy}{dx}$ අගය ආදේශයෙන්,

$$\frac{dy}{dx}(2x-y) = 1 + \left(\frac{x+2y}{2x-y}\right)^2$$

$$(2x-y)\frac{d^2y}{dx^2} = \frac{(2x-y)^2+(x+2y)^2}{(2x-y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{4x^2-4xy+y^2+x^2+4xy+4y^2}{(2x-y)^3} = \frac{5(x^2+y^2)}{(2x-y)^3}$$ //

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02. $y = e^{2m\cos^{-1}x}$ x විෂයයෙන් අවකලනයෙන්,

$$\frac{dy}{dx} = e^{2m\cos^{-1}x}\frac{d}{dx}(2m\cos^{-1}x) = y \times 2m\frac{-1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\frac{dy}{dx} = -2my \text{ ——— (1)}$$

x විෂයයෙන් අවකලනයෙන්, $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{1}{2}(1-x^2)^{\frac{-1}{2}}\frac{d}{dx}(1-x^2)\frac{dy}{dx} = -2m\frac{dy}{dx}$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = -2m\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -2m\sqrt{1-x^2}\frac{dy}{dx}$$

(1) න් ආදේශයෙන්, $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -2m \times -2my$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 4m^2y = 0$$

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03. $y = e^{m\tan^{-1}x^2}$ ——— (1)

$\frac{dy}{dx} = e^{m\tan^{-1}x^2}\frac{d}{dx}(m\tan^{-1}x^2)$ (1) න් ආදේශයෙන්,

$$\frac{dy}{dx} = y \times m\frac{1}{1+x^4}\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = \frac{2mxy}{1+x^4} \text{ ——— (2)}$$

$$(1+x^4)\frac{dy}{dx} = 2mxy$$

(1) න් $x = 1$ විට $y = e^{m\tan^{-1}1} = e^{\frac{m\pi}{4}}$

$x = 1$ විට $2\left(\frac{dy}{dx}\right)_{x=1} = 2m \times 1e^{\frac{m\pi}{4}}$

$$\left(\frac{dy}{dx}\right)_{x=1} = me^{\frac{m\pi}{4}}$$

(2) න් $(1+x^4)\frac{dy}{dx} = 2mxy$

$$(1+x^4)\frac{d^2y}{dx^2} + \frac{dy}{dx}4x^3 = 2mx\frac{dy}{dx} + y \times 2m$$

$$(1+x^4)\frac{d^2y}{dx^2} + 2x(2x^2-m)\frac{dy}{dx} - 2my = 0$$

$x = 1$, $m = 4$ විට,

$$2\left(\frac{d^2y}{dx^2}\right)_{x=1} + 2(2-4)4e^\pi - 2 \times 4e^\pi = 0$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 8e^\pi + 4e^\pi = 12e^\pi$$

04. (i) $y = x\cos\frac{1}{x}$ ——————(1) x විෂයයෙන් අවකලනයෙන්,

$$\frac{dy}{dx} = x\left(-\sin\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) + \cos\frac{1}{x}\times 1 = \frac{1}{x}\sin\frac{1}{x} + \cos\frac{1}{x}$$

x විෂයයෙන් අවකලනයෙන්, $\frac{d^2y}{dx^2} = \frac{1}{x}\cos\left(\frac{1}{x}\right)\frac{-1}{x^2} + \sin\frac{1}{x}\left(\frac{-1}{x^2}\right) - \sin\frac{1}{x}\left(\frac{-1}{x^2}\right)$

$$x^3\frac{d^2y}{dx^2} = -\cos\frac{1}{x} - x\sin\frac{1}{x} + x\sin\frac{1}{x}$$

$x^4\frac{d^2y}{dx^2} = -x\cos\frac{1}{x}$ (1) න් ආදේශයෙන්, $x^4\frac{d^2y}{dx^2} + y = 0$

(ii) $x = \cos^4 t$ t විෂයයෙන් අවකලනයෙන්,

$$\frac{dx}{dt} = 4\cos^3 t\frac{d}{dx}(\cos t) = 4\cos^3 t\sin t \text{ ——————(1)}$$

$y = \sin^4 t$ $\frac{dy}{dx} = 4\sin^3 t\frac{d}{dx}(\sin t) = 4\sin^3 t\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{4\sin^3 t\cos t}{-4\cos^3 t\sin t} = -\tan^2 t \text{ ——————(2)}$$

x විෂයයෙන් අවකලනයෙන්, $\frac{d^2y}{dx^2} = -2\tan t\sec^2 t\frac{dt}{dx} = \frac{-2\sin t}{\cos^3 t}\times\frac{dt}{dx}$

(1) න්, $\frac{dt}{dx} = \frac{-1}{4\cos^3 t\sin t}$ බව ආදේශයෙන්,

$$\frac{d^2y}{dx^2} = \frac{-2\sin t}{\cos^3 t}\times\frac{-1}{4\cos^3 t\sin t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}\sec^6 t \quad 2\frac{d^2y}{dx^2} - (\sec^2 t)^3 = 0$$

$$2\frac{d^2y}{dx^2} - (1+\tan^2 t)^3 = 0$$

(2) $\tan^2 t = \frac{dy}{dx}$ බව ආදේශයෙන්

$$2\frac{d^2y}{dx^2} - \left(1-\frac{dy}{dx}\right)^3 = 0$$

ගැටළු - 01

01. m නියත විට $y = e^{m\tan^{-1}x^2}$ නම් $(1+x^4)\frac{dy}{dx} = 2mxy$ බව පෙන්වා $x=1$ විට $\frac{dy}{dx}$ අගය ලබාගන්න. එනයින් $(1+x^4)\frac{d^2y}{dx^2} + 2x(2x^2-m) - 2my = 0$ බව පෙන්වන්න. $m=4$ විට $\left.\frac{d^2y}{dx^2}\right|_{x=1}$ අගයද ලබාගන්න.

02. $x-a>0$ විට $y = (\log_e(x-a)^2$ නම් $\frac{dy}{dx}$ සොයා $(x-a)^2\frac{d^2y}{dx^2} + (x-a)\frac{dy}{dx} = 2$ බව පෙන්වන්න. $x=2a$ විට, $\frac{d^2y}{dx^2}$ අගය සොයන්න.

03. $y = \frac{\cos^{-1}2x}{\sqrt{1-4x^2}}$ නම්, $(1-4x^2)\frac{d^2y}{dx^2} - 12x\frac{dy}{dx} - 4y = 0$ බව පෙන්වන්න.

04. $y = e^{2m\cos^{-1}}$ නම්, $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 4m^2y = 0$ බව පෙන්වන්න.

05. $y = e^x(\sin x - \cos x)$ නම්, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ බව පෙන්වන්න.

$x = \frac{\pi}{3}$ විට, ප්‍රතිඵලය සත්‍යාපනය කරන්න.

පිළිතුරු - 01

01. $y = e^{m\tan^{-1}x^2}$ —— (1)

$\frac{dy}{dx} = e^{m\tan^{-1}x^2}\frac{d}{dx}(m\tan^{-1}x^2)$

(1) න් ආදේශයෙන්,

$\frac{dy}{dx} = y \times m\frac{1}{1+x^4}\frac{d}{dx}(x^2)$

$\frac{dy}{dx} = \frac{2mxy}{1+x^4}$ ——(2)

$(1+x^4)\frac{dy}{dx} = 2mxy$

(1) න් $x=1$ විට $y = e^{m\tan^{-1}1}$

$= e^{\frac{m\pi}{4}}$

$x=1$ විට $2\left.\frac{dy}{dx}\right|_{x=1} = 2m \times 1e^{\frac{m\pi}{4}}$

$\left.\frac{dy}{dx}\right|_{x=1} = me^{\frac{m\pi}{4}}$

(2) න් $(1+x^4)\frac{dy}{dx} = 2mxy$

$(1+x^4)\frac{d^2y}{dx^2} + \frac{dy}{dx}4x^3 = 2mx\frac{dy}{dx} + y \times 2m$

$(1+x^4)\frac{d^2y}{dx^2} + 2x(2x^2-m)\frac{dy}{dx} - 2my = 0$

$x=1, m=4$ විට $2\left.\frac{d^2y}{dx^2}\right|_{x=1} + 2(2-4)4e^{\pi} - 2\times 4e^{\pi} = 0$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 8e^{\pi} + 4e^{\pi} = 12e^{\pi}$

02. $y = [\log_e(x-a)]^2$

$\frac{dy}{dx} = 2\log_e(x-a)\frac{d}{dx}\log_e(x-a)$

$= \frac{2}{x-a}\log_e(x-a)$ ——(1)

$(x-a)\frac{dy}{dx} = 2\log_e(x-a)$

$(x-a)\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{2}{x-a}$

$(x-a)^2\frac{d^2y}{dx^2} + (x-a)\frac{dy}{dx} = 2$ ——(2)

(1) න් $x=2a$ විට, $\left.\frac{dy}{dx}\right|_{x=2a} = \frac{2}{a}\log_e a$

(2) න් $(2a-a)^2\left.\frac{d^2y}{dx^2}\right|_{x=2a}+(2a-a)\frac{2}{a}\log_e a=2$ $\quad a^2\left.\frac{d^2y}{dx^2}\right|_{x=2a}=2-2\log_e a$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2a}=\frac{2(1-\log_e a)}{a^2}$$

03. $y=\frac{\cos^{-1}2x}{\sqrt{1-4x^2}}$

හරස් ගුණිතයෙන්,

$\sqrt{1-4x^2}\,y=\cos^{-1}2x$ ——(1)

$(1-4x^2)y^2=(\cos^{-1}2x)^2$

x විෂයයෙන් අවකලනයෙන්,

$(1-4x^2)2y\frac{dy}{dx}+y^2(-8x)=2\cos^{-1}2x\,\frac{-1}{\sqrt{1-4x^2}}\times 2$

(1) න් $\cos^{-1}2x=\sqrt{1-4x^2}\,y$ බව ආදේශයෙන්,

$(1-4x^2)y\frac{dy}{dx}-4xy^2=\frac{-2}{\sqrt{1-4x^2}}\sqrt{1-4x^2}\,y$

$(1-4x^2)\frac{dy}{dx}-4xy+2=0$

x විෂයයෙන් අවකලනයෙන්,

$(1-4x^2)\frac{d^2y}{dx^2}+\frac{dy}{dx}(-8x)-4x\frac{dy}{dx}-y\times 4+0=0$

$(1-4x^2)\frac{d^2y}{dx^2}-12x\frac{dy}{dx}-4y=0$

04. $y=e^{2m\cos^{-1}x}$

x විෂයයෙන් අවකලනයෙන්,

$\frac{dy}{dx}=e^{2m\cos^{-1}x}\frac{d}{dx}(2m\cos^{-1}x)$

$=y\times 2m\frac{-1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\,\frac{dy}{dx}=-2my$ —— (1)

x විෂයයෙන් අවකලනයෙන්,

$\sqrt{1-x^2}\,\frac{d^2y}{dx^2}+\frac{1}{2}(1-x^2)^{\frac{-1}{2}}\frac{d}{dx}(1-x^2)\frac{dy}{dx}=-2m\frac{dy}{dx}$

$\sqrt{1-x^2}\,\frac{d^2y}{dx^2}-\frac{x}{\sqrt{1-x^2}}\frac{dy}{dx}=-2m\frac{dy}{dx}$

$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-2m\sqrt{1-x^2}\,\frac{dy}{dx}$

(1) න් ආදේශයෙන්,

$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-2m\times -2my$

$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-4m^2y=0$

05. $y=e^x(\sin x+\cos x)$ ——(1)

x විෂයයෙන් අවකලනයෙන්,

$\frac{dy}{dx}=e^x\frac{d}{dx}(\sin x+\cos x)+(\sin x+\cos x)\frac{d}{dx}e^x$

$=e^x(\cos x-\sin x)+(\sin x+\cos x)e^x$

$\frac{dy}{dx}=2\cos x\,e^x$ ——(2)

x විෂයයෙන් අවකලනයෙන්,

$\frac{d^2y}{dx^2}=2e^x(-\sin x)+\cos x\,2e^x$

$=2e^x(\cos x-\sin x)$ ——(3)

(3) − (2) × 2 + (1) × 2 ,

$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=e^x[2\cos x-2\sin x-4\cos x+2\sin x+2\cos x]$

$=0$

$x=\frac{\pi}{3}$ විට $y=e^{\frac{\pi}{3}}\left(\sin\frac{\pi}{3}+\cos\frac{\pi}{3}\right)$

$=\frac{e^{\frac{\pi}{3}}}{2}\left(1+\sqrt{3}\right)$

$\frac{dy}{dx}=2\cos\frac{\pi}{3}e^{\frac{\pi}{3}}=e^{\frac{\pi}{3}}$

$\frac{d^2y}{dx^2}=2e^{\frac{\pi}{3}}\left(\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\right)$

$=e^{\frac{\pi}{3}}\left(1-\sqrt{3}\right)$, අගයන් ආදේශයෙන්

$e^{\frac{\pi}{3}}\left(1-\sqrt{3}\right)-2e^{\frac{\pi}{3}}+2\frac{e^{\frac{\pi}{3}}}{2}\left(1+\sqrt{3}\right)=0$

$e^{\frac{\pi}{3}}\left(1-\sqrt{3}-2+1+\sqrt{3}\right)=0$

$0=0$

ප්‍රතිඵලය සත්‍ය වේ.

පහත සඳහන් ශ්‍රිත සඳහා දී ඇති සම්බන්ධතා සාධනය කරන්න.

01. $y = \text{Sin}(\text{Sin}\, x)$ විට $\frac{d^2y}{dx^2} + \text{Tan}\, x \frac{dy}{dx} + y\,\text{Cos}^2 x = 0$

02. $y = e^{\text{Tan}\, x}$ විට $\frac{d^2y}{dx^2} - \frac{1 + \text{Sin}\, 2x}{\text{Cos}^2 x} \frac{dy}{dx} = 0$

03. $y = \text{In}\,|3x + 1|$ විට $(3x + 1) \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} = 0$

04. $y = \frac{x}{1 - x}$ විට $(1 - x) \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} = 0$

05. $y = In\,|\text{Sin}\, x|$ විට $\frac{d^2y}{dx^2} + \text{Col}\, x \frac{dy}{dx} + 1 = 0$

06. $y = x\sqrt{1 - x}$ විට $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$

07. $y = xe^x$ විට $\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x$

08. $y = \frac{1}{x}\text{Sin}\, x$ විට $x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} yx = 0$

09. $y = \text{Sin}(a\,\text{Cos}\, x)$ විට $\frac{d^2y}{dx^2} - \text{Cot}\, x \frac{dy}{dx} - yx^2\,\text{Sin}^2 x = 0$

10. $y = \text{Tan}^{-1}(x^2)$ විට $(1 + x^4) \frac{d^2y}{dx^2} + 4x^3 \frac{dy}{dx} = 0$

11. $y = e^{ax}\,\text{Cos}\, bx$ විට $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)\, y = 0$

12. $y = (\text{Sin}^{-1} 2x)^2$ විට $(1 - 4x^2) \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} = 8$

13. $y = \frac{\text{Sin}^{-1} x}{\sqrt{1 - x^2}}$ විට $(1 - x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$

14. $y = [\text{In}\,|x - a|\,]^2$ විට $(x - a)^2 \frac{d^2y}{dx^2} + (x - a) \frac{dy}{dx} = 2$

15. $y = x\,\text{Cos}\left(\frac{1}{x}\right)$ විට $x^4 \frac{d^2y}{dx^2} + y = 0$

16. $y = \text{Sin}\,[a\,\text{Sin}^{-1}(mx)]$ විට $[1 - (mx^2)] \frac{d^2y}{dx^2} - mx^2 \frac{dy}{dx} + am^2y = 0$

17. $y = be^x + ce^{2x}$ විට $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$

18. $y = A\,\text{Cos}\,(\text{In}\,|x|) + B\,\text{Sin}\,(\text{In}\,|x|)$ විට $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

19. $y = x^n e^{-y}$ විට $(1 + y)^2 x \frac{d^2y}{dx^2} + [(1 + y)^2 - n] \frac{dy}{dx} = 0$

20. $x^2 y = A + \ln|2+x|$ විට $[x(2+x)^2 \frac{d^2y}{dx^2} + 4x(2+x)\frac{dy}{dx} + 2y(2+x) + 1 = 0$

21. $y = \ln|x^2y|$ විට $x^2y(y-1)\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^2 + 2y^2 = 0$

22. $x = a\,\text{Cost}$ $y = a\,\text{Sint}$ විට $y\frac{d^2y}{dx^2} + \frac{dy^2}{dx} + 1 = 0$

23. $x = 2(t - \text{Cost})$ හා $y = 0(1 + \text{Sint})$ විට $y^2\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^{2+y} = 2$

24. $x = 2t^3 + 1$, $y = 4t^4 - 1$ නම් $\left(\frac{dy}{dx}\right)\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 = 0$

25. $x^2 + y^2 = t - \frac{1}{t}$ හා , $x^4 + y^4 = t^2 + \frac{1}{t^2}$ විට $\frac{d^2y}{dx^2} + \frac{2dy}{dx} = 0$

පිළිතුරු - 02

01. $y = Sin\,(Sinx)$

$\frac{dy}{dx} = Cos\,(Sinx)Cosx$

නැවත x විෂයෙන් අවකලනයෙන්,

$\frac{d^2y}{dx^2} = Cos(Sinx)(-Sinx) + Cosx\,[-Sin(Sinx)]Cosx$

$= Cos(Sinx)Cosx\,(-\frac{Sinx}{Cosx}) - Sin(Sinx)\,Cos^2x$

$\frac{d^2y}{dx^2} = \frac{dy}{dx}(-Tanx) - y\,Cos^2x$

$\frac{d^2y}{dx^2} + Tanx\frac{dy}{dx} + yCos^2x = 0$

02. $y = e^{Tanx}$

$\frac{dy}{dx} = e^{Tanx}\,Sec^2x$

$= \frac{e^{Tanx}}{Cos^2x}$

$Cos^2x\frac{dy}{dx} = e^{Tanx}$

නැවත x විෂයෙන් අවකලනයෙන්,

$Cos^2x\frac{d^2y}{dx^2} - 2CosxSinx\frac{dy}{dx} = e^{Tanx}\,Sec^2x$

$\frac{d^2y}{dx^2} - \frac{2\,Sinx}{Cos\,x}\frac{dy}{dx} = e^{Tanx}\,Sec^2x \times \frac{1}{Cos^2x} = \frac{1}{Cos^2x}\frac{dy}{dx}$

$\frac{d^2y}{dx^2} - \frac{dy}{dx}\left[\frac{2Sinx}{Cosx} + \frac{1}{Cos^2x}\right] = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx}\left[\frac{2SinxCosx+1}{Cos^2x}\right] = 0$

$\frac{d^2y}{dx^2} - \left(\frac{1+Sin2x}{Cos^2x}\right)\frac{dy}{dx} = 0$

03. $y = \ln|3x+1|$

$\frac{dy}{dx} = \frac{1}{3x+1} \times 3$

$(3x+1)\frac{dy}{dx} = 3$

x විෂයෙන් අවකලනයෙන්,

$(3x+1)\frac{d^2y}{dx^2} + \frac{dy}{dx}(3) = 0$

$(3x+1)\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$

04. $y = \frac{x}{1-x}$

$\frac{dy}{dx} = \frac{(1-x)1 - x(-1)}{(1-x)^2}$

$(1-x)^2\frac{dy}{dx} = 1$

x විෂයෙන් අවකලනයෙන්,

$(1-x)^2\frac{d^2y}{dx^2} + 2(1-x)(-1)\frac{dy}{dx} = 0$

$(1-x)^2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$

05. $y = \ln|Sinx|$

$\frac{dy}{dx} = \frac{1}{Sinx}\,Cosx$

$Sin\,x\frac{dy}{dx} = Cosx$

x විෂයෙන් අවකලනයෙන්,

$Sinx\frac{d^2y}{dx^2} + \frac{dy}{dx}Cosx = -Sinx$

$\frac{d^2y}{dx^2} + Cotx\frac{dy}{dx} + 1 = 0$

06. $y = x\sqrt{1-x^2}$

$\frac{dy}{dx} = \sqrt{1-x^2} + \frac{x(-2x)}{2\sqrt{1-x^2}}$

$\sqrt{1-x^2}\frac{dy}{dx} = (1-x^2) - x^2$

$\sqrt{1-x^2}\frac{dy}{dx} = 1 - 2x^2$

$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{(-2x)}{2\sqrt{1-x^2}} = -4x$

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -4x\sqrt{1-x^2}$

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$

07. $y = x\,e^x$

$\frac{dy}{dx} = x\,e^x + e^x$

$\frac{d^2y}{dx^2} = x\,e^x + e^x + e^x$

$\frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x$

08. $y = \frac{1}{x} Sinx$

$yx = Sinx$

$y+x\frac{dy}{dx} = Cosx$

$\frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -Sinx$

$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -Sinx$

$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} = -\frac{1}{x} Sinx$

$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y = 0$

09. $y = Sin(a\ Cosx)$

$\frac{dy}{dx} = Cos\ (aCosx)(-aSinx)$

$\frac{d^2y}{dx^2} = -Sin(aCosx)(-aSinx)^2 + Cos(aCosx)(-aCosx)$

$\frac{d^2y}{dx^2} = -Sin(aCosx)(a^2Sin^2x) + Cos(aCosx)(-aSinx)\frac{(-aCosx)}{-aSinx}$

$= -ya^2Sin^2x + \frac{dy}{dx} Cotx$

$\frac{d^2y}{dx^2} - Cotx\frac{dy}{dx} + ya^2Sin^2x = 0$

10. $y = Tan^{-1}(x^2)$

$\frac{dy}{dx} = \frac{1}{1+(x^2)^2} 2x$

$(1+x^4)\frac{dy}{dx} = 2x$

$(1+x^4)\frac{d^2y}{dx^2} + 4x^3\frac{dy}{dx} = 2$

11. $y = e^{ax}Cosbx$

$\frac{dy}{dx} = e^{ax}(-Sinbx)b + Cosbx(ae^{ax})$

$= e^{ax}(aCosbx + bSinbx)$

$\frac{dy}{dx} = e^{ax}aCosbx - be^{ax}Sinbx$

$= ay - be^{ax}Sinbx$

$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - b[ae^{ax}Sinx + be^{ax}Cosbx]$

$= a\frac{dy}{dx} - a[be^{ax}Sinbx] - b^2e^{ax}Cosbx$

$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - a[ay - \frac{dy}{dx}] - b^2y$

$= 2a\frac{dy}{dx} - a^2y - b^2y$

$= 2a\frac{dy}{dx} - (a^2+b^2)y$

$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)\ y = 0$

12. $y = (Sin^{-1}2x)^2$

$\frac{dy}{dx} = 2Sin^{-1}2x \quad \frac{1}{\sqrt{1-4x^2}} \times 2$

$\sqrt{1-4x^2}\ \frac{dy}{dx} = 4(Sin^{-1}2x)$

$\sqrt{1-4x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{1}{2} \frac{(-8x)}{\sqrt{1-4x^2}} = \frac{8}{\sqrt{1-4x^2}}$

$(1-4x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} = 8$

13. $y = \frac{Sin^{-1}x}{\sqrt{1-x^2}}$

$y\sqrt{1-x^2} = Sin^{-1}x$

$y^2(1-x^2) = (Sin^{-1}x)^2$

x විෂයෙන් අවකලනයෙන්,

$y^2(-2x) + (1-x^2)2y\frac{dy}{dx} = \frac{2Sin^{-1}x}{\sqrt{1-x^2}}$

$y^2(-2x) + (1-x^2)2y\frac{dy}{dx} = 2y$

$(1-x^2)\frac{dy}{dx} - yx = 1$

$(1-x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) - y - x\frac{dy}{dx} = 0$

$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

14. $y = [ln|x-a|]^2$

$\frac{dy}{dx} = 2ln|x-a| \times \frac{1}{x-a}$

$(x-a)\frac{dy}{dx} = 2\ ln|x-a|$

නැවත x විෂයෙන් අවකලනයෙන්,

$(x-a)\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{2}{x-a}$

$(x-a)^2\frac{d^2y}{dx^2} + (x-a)\frac{dy}{dx} = 2$

15. $y = x\ Cos(\frac{1}{x})$

$\frac{dy}{dx} = xSin(\frac{1}{x})(-\frac{1}{x^2}) + Cos(\frac{1}{x})$

$= \frac{1}{x}Sin(\frac{1}{x}) + Cos(\frac{1}{x})$

$x\frac{dy}{dx} = Sin(\frac{1}{x}) + xCos(\frac{1}{x})$

$x\frac{dy}{dx}\ Sin\ (\frac{1}{2}) + y$

නැවත x විෂයෙන් අවකලනයෙන්,

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = Cos(\frac{1}{x})(-\frac{1}{x^2}) + \frac{dy}{dx}$

$x\frac{d^2y}{dx^2} = -\frac{1}{x^2}Cos(\frac{1}{x})$

$x^4\frac{d^2y}{dx^2} = -xCos\frac{1}{x}$

$x^4\frac{d^2y}{dx^2} + y = 0$

16. $y = Sin[aSin^{-1}(mx)]$

$\frac{dy}{dx} = Cos[asin^{-1}(mx)] \times \frac{a(m)}{\sqrt{1-(mx)^2}}$

$\sqrt{1-(mx)^2}\ \frac{dy}{dx} = amCos[asin^{-1}(mx)]$

නැවත x විෂයෙන් අවකලනයෙන්,

$\sqrt{1-(mx)^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1(-2mx)m}{2\sqrt{1-(mx)^2}}$

$= -amSin[asin^{-1}(mx)]\frac{am}{\sqrt{1-(mx)^2}}$

$[1-(mx)^2]\frac{d^2y}{dx^2} - m^2x\frac{dy}{dx}$

$= -am^2Sin[asin^{-1}(mx)]$

$[1-(mx)^2]\frac{d^2y}{dx^2} - mx\frac{dy}{dx} = -(am)^2\ y$

$[1-(mx)^2]\frac{d^2y}{dx^2} - mx\frac{dy}{dx} + (am)^2y = 0$

17. $y=be^x+ce^{2x}$

$\frac{dy}{dx}=be^x+2ce^{2x}$

නැවත x විෂයෙන් අවකලනයෙන්,

$\frac{d^2y}{dx^2}=be^x+2c\ (2e^{2x})$

$\frac{d^2y}{dx^2}=(be^x+2ce^{2x})+2ce^{2x}$

$=3(be^x+2ce^{2x})-2be^x-2ce^{2x}$

$=3\frac{dy}{dx}-2be^x-2ce^{2x}$

$=3\frac{dy}{dx}-2[be^x+ce^{2x}]$

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$

18. $y=ACos(lnx)+BSin(lnx)$

$\frac{dy}{dx}=-ASin(lnx)\frac{1}{x}+BCos[lnx]x\frac{1}{x}$

$x\frac{dy}{dx}=-ASin(lnx)+BCos[lnx]$

නැවත x විෂයෙන් අවකලනයෙන්,

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-[ACos(lnx)+Sin(lnx)\ B]$

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$

19. $y=x^ne^y$

$\frac{dy}{dx}=nx^{n-1}e^y+x^ne^y\frac{dy}{dx}$

x න් ගුණ කිරීමෙන්,

$x\frac{dy}{dx}=nx^ne^y-\frac{dy}{dx}(x^ne^y)x$

$x\frac{dy}{dx}=ny-xy\frac{dy}{dx}$

$x(1+y)\frac{dy}{dx}=ny$

$x\frac{dy}{dx}=\frac{ny}{1+y}$

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{[(1+y)n-ny]}{(1+y)^2}\frac{dy}{dx}$

$(1+y)^2x\frac{d^2y}{dx^2}+(x+y)^2\frac{dy}{dx}=\frac{dy}{dx}n$

$(1+y)^2x\frac{d^2y}{dx^2}+[(1+y)^2-n]\frac{dy}{dx}=0$

20. $x^2y=A+ln(2+x)$

$x^2\frac{dy}{dx}+y2x=\frac{1}{2+x}$

නැවත x විෂයෙන් අවකලනයෙන්,

$2x\frac{dy}{dx}+x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+2y=\frac{-1}{(2+x)^2}$

$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\frac{-1}{(2+x)^2}$

$(2+x)^2x^2\frac{d^2y}{dx^2}+(2+x)^2\ 4x\frac{dy}{dx}+2(2+x)^2\ y+1=0$

21. $y=ln|x^2y|$

$=lnx^2+lny$

$y=2lnx+lny$

$\frac{dy}{dx}=\frac{2}{x}+\frac{1}{y}\frac{dy}{dx}$, $\frac{y-1}{y}\frac{dy}{dx}=\frac{2}{x}$

නැවත x විෂයෙන් අවකලනයෙන්,

$\frac{y-1}{y}\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{1}{y^2}=\frac{-2}{x^2}$

$(y-1)x^2y\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+2y^2=0$

22. $x=aCost$ නම් $\frac{dx}{dt}=-aSint$

$y=aSint$ නම් $\frac{dy}{dt}=aCost$

$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ නිසා

$=\frac{dy}{dt}\times\frac{1}{dx/dt}=aCost\times\frac{-1}{aSint}\Rightarrow\frac{dy}{dx}=-\frac{x}{y}\Rightarrow y\frac{dy}{dx}=-x$

නැවත x විෂයෙන් අවකලනයෙන්,

$y\frac{d^2y}{dx^2}+(\frac{dy}{dx})(\frac{dy}{dx})=-1$

$y\frac{d^2y}{dx^2}+(\frac{dy}{dx})^2+1=0$

23. $x=2(t-Cost)$ නම් $\frac{dx}{dt}=2(1+Sint)$

$y=2(1+Sint)$ නම් $\frac{dy}{dt}=2Cost$

$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ නිසා

$=2Cost\times\frac{1}{2(1+Sint)}\Rightarrow=\frac{2Cost}{y}$

$y\frac{dy}{dx}=2Cost\Rightarrow y\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{dy}{dx}=-2Sint\frac{dt}{dx}$

$y\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{dy}{dx}=\frac{-2Sint}{2(1+Sint)}=\frac{-2Sint}{y}$

$y^2\frac{d^2y}{dx^2}+y(\frac{dy}{dx})^2=-2Sin\ t=2-y$

24. $x=2t^3+1$ වේ $\frac{dx}{dt}=6t^2$

$y=4t^4-1$ වේ $\frac{dy}{dt}=16t^3$

$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ නිසා

$=16t^3\times\frac{1}{6t^2}=\frac{8t}{3}$

$\frac{d^2y}{dx^2}=\frac{d}{dx}(\frac{dy}{dx})=\frac{d}{dx}(\frac{8t}{3})$

$=\frac{d}{dx}(\frac{8t}{3})\times\frac{dt}{dx}\Rightarrow\frac{d^2y}{dx^2}=\frac{8}{3}\times\frac{1}{6t^2}\Rightarrow\frac{d^2y}{dx^2}=\frac{4}{9t^2}$

$\frac{d^3y}{dx^3}=\frac{d}{dx}(\frac{d^2y}{dx^2})=\frac{d}{dx}(\frac{4}{9t^2})=\frac{d}{dt}(\frac{4}{9t^2})\times\frac{dt}{dx}$

$=\frac{4}{9}(-\frac{2}{t^3})\times\frac{1}{6t^2}=\frac{-8}{54t^5}$

$\frac{dy}{dx}\frac{d^3y}{dx^3}=\frac{-8}{54t^5}\times\frac{8t}{3}=\frac{-32}{81t^4}=-2(\frac{4}{9t^2})^2=-2(\frac{d^2y}{dx^2})^2$

$\frac{dy}{dx}\frac{d^3y}{dx^3}+2(\frac{d^2y}{dx^2})^2=0$

25. $x^2+y^2=t-\frac{1}{t}$ හා $x^4+y^4=t^2+\frac{1}{t^2}$ නම්

$(x^2+y^2)^2=(t-\frac{1}{t})^2=t^2+\frac{1}{t^2}-2$

$x^4+2y^2x^2+y^4=x^4+y^4-2$

$2x^2y^2=-2$

x විෂයෙන් අවකලනයෙන්,

$2xy^2+2yx^2\frac{dy}{dx}=0$

$y+x\frac{dy}{dx}=0$

නැවත x විෂයෙන් අවකලනයෙන්,

$\frac{dy}{dx}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

$\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$